Industrial Automation

Reliability and Availability Exercises

In our running example, we consider a chain of Christmas lights consisting of many light bulbs.

1. Let us first consider a single light bulb.
   (a) Under what conditions can we assume that a single light bulb's failure rate is constant?
   (b) From now on, let us assume that the failure rate of a single light bulb is indeed constant. Moreover, assume that a light bulb is expected to burn out after 1000 hours. What is its failure rate?
   (c) What is the reliability of a single light bulb?
2. We consider a chain of Christmas lights consisting of 50 serially connected light bulbs. This means that if any light bulb burns out, the electric circuit is interrupted and all light bulbs go off.
   (a) What is the reliability of the whole chain?
   (b) How long can we expect the light chain to work?
3. We now assume that the light bulbs are connected in parallel. Thus, if a light bulb burns out, the others may continue operating. We consider a light chain to be working as long as not more than 10 light bulbs are burned out.
   (a) What is the reliability of the whole chain?
   (b) How long can we expect the light chain to work? (No need to calculate the value, just give a mathematical expression describing it.)
4. We assume that during the Christmas period, the light chain is used every evening for 4 hours during the whole month of December. Answer the following questions for both the serial and the parallel variant of the light chain.
   (a) What is the probability that the light chain will remain functional during the whole Christmas period? (Feel free to serve yourself with mathematical tools like WolframAlpha to compute the resulting numbers.)
   (b) What is the probability that the light chain survives the Christmas periods of two consecutive years?

(c) If, after the first year's Christmas period, 5 light bulbs burn out, what is the probability of the light chain surviving the next Christmas period? (Only relevant for the parallel case.)

(d) Assume that the package with the light chain contains 2 replacement light bulbs. How do the answers to the 3 above questions change?

(e) What are the answers to the previous 4 questions if we use LED light bulbs with an expected life time of 5000 hours?

5. Now consider a case of a light chain that is supposed to be on permanently. If a light bulb goes off, it takes 1 hour to be noticed and replaced on expectation. For simplicity, we assume that repairs happen independently and can even be carried out in parallel.

(a) What is the (stationary) availability of the serial variant of the light chain?

(b) Draw a Markov chain modeling the reliability of both the serial and the parallel variant of the light chain. For simplicity, assume that in the parallel case, the light chain goes off automatically when more than 10 light bulbs burn out.

(c) Set up the differential equations describing the states of the Markov chains.

(d) What are the initial conditions of the differential equations for a new light chain (i.e. when none of the light bulbs burned out yet)?

(e) How would you calculate the mean time to failure (MTTF) of the light chains? (Describe the steps, without performing the actual calculation.)

(f) How would you calculate the availability of the light chains? (Describe the steps, without performing the actual calculation.)