## Week $2-C++$ starter: Manipulating floating point numbers

The goal of the present exercises is to manipulate the floating point numbers in $\mathrm{C}++$

## Exercise 1: Casting integers and float

- Execute the following program:

```
int a = 0.5;
std::cout << a << std::endl;
```

- Launch the program and describe what you observe. How could this be avoided.
- Execute now

```
double a = 1/2;
std::cout << a << std::endl;
```

- Please explain again what happened.
- Can this happen in Python language?


## Exercise 2: Overflow

- The exponential function can be computed using the following function:

```
double x = 10.;
double a = exp(x);
std::cout << a << std::endl;
```

(use the manual if you seek for information about the exp function)

- Create a function which computes the following hyperbolic tangent:

$$
\begin{equation*}
\tanh (x)=\frac{\exp (x)-\exp (-x)}{\exp (x)+\exp (-x)} \tag{1}
\end{equation*}
$$

- Make a loop to compute 10 points of the curve $\tanh (x)$ for $x \in[0,1000]$.
- Try to factor the $\exp (x)$ to find an expression avoiding the overflow.


## Exercise 3: Underflow

- Find (and program) the best way to evaluate the function

$$
\begin{equation*}
\tanh (x)-1=\frac{\exp (x)-\exp (-x)}{\exp (x)+\exp (-x)}-1 \tag{2}
\end{equation*}
$$

- Let us now consider the motion of two photons moving at the velocity $v=3 \cdot 10^{8} \mathrm{~m} / \mathrm{s}$ in the same direction. Their position evolution can be computed with:

$$
\begin{equation*}
p_{i}(t)=p_{i}(0)+t v \tag{3}
\end{equation*}
$$

with $p_{i}(t)(i$ being 0 or 1$)$ their position at time $t$.
If we discretize time we can provide the time integration scheme:

$$
\begin{equation*}
p_{i}^{n+1}=p_{i}^{n}+d t \cdot v \tag{4}
\end{equation*}
$$

- Program the function

```
double evolution(double pn, double v, double dt);
```

which returns the new position provided the position at the previous step ( pn ), the velocity (v) and the time step (dt)

- Make a program which sets the initial position of the two photons being $p_{0}(0)=0$ and $p_{1}(0)=10^{-3}$. Thus they are distant by one millimeter.
- Then compute the positions evolution during 40 time steps with $d t=1000$ seconds (less than 1 hour).
- Compute the distance $p_{1}(t)-p_{0}(t)$. Explain what you see.

